Small Satellite LEO Maneuvers with Low-Power Electric Propulsion

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Chemical propulsion currently provides the means for near-Earth satellite orbit maneuvers. As electric propulsion (EP) devices become more accepted and flight qualified, EP may enable increases in satellite payload mass while decreasing satellite propellant mass. The Air Force Operationally-Responsive Space (ORS) program proposes the use of commercially available and readily configurable satellite buses with masses of approximately 500 kg for near-Earth missions. This paper discusses three orbital maneuvers considered important in the characterization of EP use for satellites in near-Earth orbits. The first maneuver changes an orbit from low-Earth orbit (LEO) at 800 km to medium-Earth orbit (MEO) at 20,000 km. The second maneuver changes the inclination of an orbit at LEO by 90°, while the third maneuver rephases a satellite in LEO orbit by 180°. Each maneuver considers thruster specific impulse, I_{sp}, from 1,000 to 3,000 s and thruster power from 100 W to 1.5 kW for a 500 kg satellite to obtain propellant mass and transfer time. In general, as I_{sp} increases the transfer time increases and the propellant mass decreases. These transfer times range from hours to years, but mission constraints will define appropriate levels of I_{sp} and power. Analysis of these maneuvers finds that EP is beneficial to near-Earth satellites for altitude and phase changes. EP is not currently beneficial to missions requiring large inclination changes, but identifies a key interest in future low-power EP devices.

Nomenclature

Δi	=	inclination change, rad
I_{sp}	=	specific impulse, s
m	=	spacecraft mass, kg
g	=	acceleration of gravity at Earth's surface, m/s^2
P	=	input power, W
t_{loiter}	=	loiter (coasting) time during phase change, s
ΔV	=	velocity change, m/s
V	=	orbital velocity, m/s
η	=	thruster efficiency
θ	=	longitude angle, rad

Subscripts:

LEO	=	low-Earth orbit
MEO	=	medium-Earth orbit

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I. Introduction

CURRENT near-Earth satellite lifetime is determined by the amount of fuel the satellite can store. Orbital maneuvers, including orbit adjustment from atmospheric drag losses, require a velocity change, ΔV, which the satellite must provide or the orbit degrades and the satellite eventually reenters Earth's atmosphere. Current chemical propulsion systems are limited by the available chemical energy which is stored as fuel mass on a spacecraft. Longer satellite lifetimes require larger amounts of fuel, but this creates trade studies in which satellite mission designers must trade between satellite lifetime, delivered payload mass, launch vehicle capability, etc. Electric propulsion (EP) is limited by the amount of available power and stored propellant to operate the thruster. Since EP has a higher I_{sp} than chemical propulsion, the lifetime fuel throughput of EP can be orders of magnitude smaller than chemical propulsion fuel throughput for a similar ΔV mission requirement.[1-5]

In past studies, Oh investigated the use of various solar electric propulsion technologies for interplanetary missions.[6] Oh et al. investigated optimal orbit transfers to geostationary Earth orbit (GEO) using combined chemical-electric stages, where the electric-stage I_{sp} was free for optimization.[7]

Current EP use is on larger satellites in GEO. Space Systems/LORAL uses the SPT-100 thruster in GEO for station keeping and altitude changes.[8] The Boeing 702 satellite uses four 25-cm XIPS thrusters for initial orbit insertion, north-south and east-west station keeping, attitude control, and momentum dumping while in GEO.[9] The Lockheed Martin and Northrop Grumman built Advanced Extremely High Frequency (AEHF) satellites use the Aerojet BPT-4000 Hall thruster for orbit raising and station keeping in GEO.[10]

This paper studies EP use and effectiveness with three orbital maneuvers in LEO. The first maneuver increases the altitude of the satellite orbit from an 800-km circular low-Earth orbit (LEO) to a 20,000-km altitude circular medium-Earth orbit (MEO). This is a common maneuver as many launch vehicles insert satellites into parking orbits before final orbit insertion. Second, the satellite's orbit inclination changes by 90°. This maneuver is essential for changing satellite coverage from one area of the Earth to another. Third, the satellite phasing is changed by 180°. Such a maneuver is useful during launch of multiple satellites for a proper satellite constellation and mission-required ground coverage.

It is important to note that this study of EP use in various orbital maneuvers does not differentiate between types of EP. Hall effect thrusters, ion engines, MPDs, and PPTs are all considered under the selected ranges of I_{sp} and input power. Commercial, low-power EP systems currently have a range of power between 100 W and 1.5 kW with an I_{sp} range of 1,000 to 3,000 s.[11-13] These thrusters can effectively change a satellite's orbit altitude, alter the inclination of a satellite's orbit, as well as modify the phase of the satellite in the orbit. The decision of which EP system to use in a mission is determined by mission constraints and objectives.

II. Low-Thrust Orbital Mechanics Computations

Computation of low-thrust trajectories is a difficult task due to the nature of long-duration spiral transfers, which are complicated by Earth-shadow eclipses and loss of input power to the thrusters. Furthermore, a general three-dimensional (3-D) transfer requires simultaneously changing and controlling the orbit's energy, eccentricity, and inclination. For the general 3-D low-thrust transfer with Earth-shadow effects, the time history of the thrust-steering direction must be judiciously selected in order to obtain the transfer which minimizes transfer time or propellant usage. Kluever and Oleson present a strategy for computing optimal low-thrust transfers for general 3-D maneuvers which has evolved into the low-thrust trajectory optimization program SEPDOC.[14] SEPDOC was recently used to obtain optimal orbit-raising maneuvers for an SEP tug for transferring a payload from LEO to a high-energy orbit.[15]

Fortunately, the low-thrust maneuvers investigated in this paper are considerably easier to compute than the transfers computed by Kluever and Oleson as well as Bonin and Kaya.[14,15] All maneuvers presented in this paper involve continuous thrust, where we assume that power can be maintained at a constant level throughout the transfer, even during Earth-shadow arcs. Secondly, the maneuvers considered in this paper either involve dedicated thrusting in the orbital plane (for an increase or decrease in altitude), or dedicated thrusting normal to the orbital plane (for plane changes). Therefore, there is no need to use a complex trajectory optimization program such as SEPDOC, and in most cases a purely analytic solution is available. These respective trajectory computations are outlined in the following subsections.

A. Altitude Change

Because the first maneuver case involves planar transfers between circular orbits with continuous thrust, we can obtain analytic solutions for the low-thrust velocity increment (ΔV) using Edelbaum's method.[16] These analytic solutions are valid for transfers between inclined circular orbits with continuous thrust. In such cases, the in-plane component of thrust acceleration is always directed along the velocity vector, and therefore the net eccentricity change per revolution is zero. For the simplified case of a planar, circle-to-circle altitude change, Edelbaum's analytic solution is

$$\Delta V = V_{\mathrm{LEO}} - V_{\mathrm{MEO}} \tag{1}$$

where V_{LEO} = 7,451.8 m/s is the velocity for an 800-km altitude circular LEO, and V_{MEO} = 3,887.3 m/s is the velocity for a circular 20,000-km altitude MEO. The final mass of the spacecraft in MEO is easily computed from fixed ΔV and the rocket equation:

$$m_{\mathrm{MEO}} = m_{\mathrm{LEO}} \exp\left(-\Delta V / g I_{\mathrm{sp}}\right) \tag{2}$$

Finally, the transfer time t_f is computed from the propellant mass and the constant mass-flow rate

$$t_f = \frac{m_{prop}}{\dot{m}} = \frac{m_{LEO} - m_{MEO}}{\dot{m}} \tag{3}$$

where the mass-flow rate is

$$\dot{m} = \frac{2\eta P}{\left(g I_{sp}\right)^2} \tag{4}$$

and η is the thruster efficiency and P is the input power.

B. Plane Change

The second maneuver involves a pure 90-deg plane (inclination) change without change in energy or eccentricity, and therefore the thrust is always directed normal to the orbital plane and switches directions at the anti-nodes. Edelbaum's analytic solution is

$$\Delta V = V_{LEO} \Delta i \frac{\pi}{2} \tag{5}$$

where Δi is the desired plane change, which in this case is $\pi/2$ rad (i.e., 90-deg). The final mass after the plane change is computed using the rocket equation (2), and the total transfer time is computed using Eqs. (3) and (4).

C. Phase Change

The third maneuver involves a phase or longitude change within the circular LEO. A numerical approach is required to accurately compute the trip time and ΔV. For a 180-deg phase change, the spacecraft initially spirals up to a higher circular orbit. The circle-to-circle spiral-up transfer is computed by using an analytic solution developed by Gao and Kluever, which is valid for tangential thrust.[17] This analytic solution determines the transfer time and semi-major axis (or radius, in this case) time histories for the spiral-up maneuver. The resulting change in phase angle between the spacecraft and the desired target location is

$$\Delta\theta = \int_0^t \left(\dot{\theta}_{LEO} - \dot{\theta}\right) dt \tag{6}$$

where $\dot{\theta}_{LEO}$ is the constant orbital rate (rad/s) of the target in LEO, and $\dot{\theta}$ is the instantaneous orbital rate of the spacecraft as it spirals up to a higher altitude. The integral in Eq. (6) is computed numerically using trapezoidal-rule integration. After the spacecraft reaches the desired higher altitude, the remaining phase-angle change is computed as $\theta_{loiter} = \pi - 2\Delta\theta$ rad, which assumes that the subsequent spiral-down transfer takes the same amount of time and produces the same relative phase change as the spiral-up transfer. The "loiter time" or coasting time spent in the higher orbit is then computed from

$$t_{loiter} = \frac{\theta_{loiter}}{\dot{\theta}_{LEO} - \dot{\theta}_f} \tag{7}$$

where $\dot{\theta}_f$ is the orbital rate of the higher (loiter) orbit. The total maneuver time is computed from the sum of the two powered arcs (spiral-up and spiral-down transfers) plus loiter time as determined by Eq. (7). Total propellant mass is computed from the (equal) transfer times for the spiral-up and spiral-down maneuvers. The re-phasing maneuver calculations are iteratively repeated for a higher loiter altitude until the loiter time becomes zero; this limiting case represents the fastest possible 180-deg phasing maneuver where the spacecraft continually spirals up during half of the entire transfer time and then spirals down during the remaining half of the transfer.

III. Data Analysis and Discussion

Data analysis requires initial estimates of satellite mission design criteria. As provided by the tentative Air Force ORS mission requirements, initial LEO satellite mass estimates at 500 kg. Thruster efficiency η is set to 0.5, which is approximately the efficiency of current commercial, low-power EP systems at the power levels under investigation. Thruster power ranges between 100 W and 1.5 kW, depending on the available bus power and additional solar arrays or secondary batteries. Finally, thruster I_{sp} ranges between 1,000 and 3,000 s. With the initial criteria selected, the final satellite mass, propellant mass, and transfer time are determined by using the respective trajectory computations for each maneuver. These results are then compared with current Air Force ORS requirements.

The first assumption made in these analyses is that transfer times over one year are not practical or useful for mission requirements, and transfer times greater than this are not studied in depth. This analysis also excludes EP component and support masses, including power processing units, solar arrays, and the thruster body. Therefore, the 500 kg initial satellite mass assumes there is no limitation on fuel mass because of the propulsion system, expected satellite payload mass, and structure mass. The remaining mass for all spacecraft subsystems except fuel is simply found by subtracting the required propellant mass from the 500 kg initial satellite mass.

A. Altitude Change

This maneuver includes an orbit altitude change from LEO (800 km) to MEO (20,000 km). As previously stated, this analysis includes power ranges of 100 W to 1.5 kW and I_{sp} ranges of 1,000 to 3,000 s. Figure 1 shows transfer time, in days, as a function of thruster power. Each line corresponds to a specific I_{sp}, held constant throughout the analysis. This shows that for a desired altitude change from 800 to 20,000 km with a satellite mass of 500 kg and thruster efficiency of 0.5, transfer times range from 113 days to 400 days (although the analysis calculates transfer times up to 5,700 days). In general, as power increases the transfer time is found to decrease. Although each mission will contain its own constraints, maneuvers that require a transfer time of more than one year are generally not considered within the ORS mission criteria. Therefore, EP with an I_{sp} of 1,000 s or more will allow a transfer time within one year, but as the I_{sp} increases, the power required to maintain this one year transfer time also increases significantly. Thrusters with an I_{sp} of 2,400 s or greater require a minimum of 1.2 kW of power. This may require more power than readily available satellite buses provide. Figure 1 also shows that, for a fixed input power, as the I_{sp} increases, the transfer time also increases. With a fixed input power, this is because increases in I_{sp} correlate with a decrease in thrust, resulting in longer transfer times.

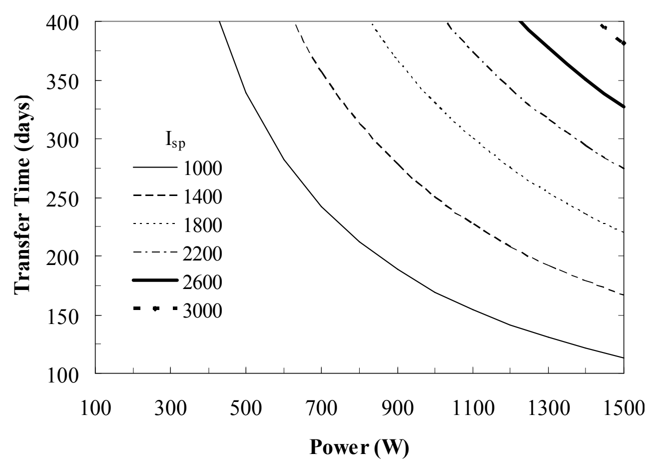

Figure 1. LEO-to-MEO transfer time as a function of thruster power and I_{sp}.

Figure 2 shows the propellant mass required by a 500 kg satellite to perform the altitude change from LEO-to-MEO. This analysis is independent of thruster power, as the calculation of propellant mass only requires initial mass and I_{sp}. The propellant mass for LEO-to-MEO orbit transfers ranges from 152 to 57 kg, with the lower mass corresponding to the higher I_{sp}. Figure 2 also shows that the propellant mass fraction (m_{prop}/m_{LEO}) ranges from approximately 0.30 to 0.11. The lower mass fraction also corresponds to the higher I_{sp}.

In summary, the propellant mass required for a maneuver from LEO-to-MEO is 30% or less of the overall satellite mass. This allows for larger payload capability with the same structural mass (provided by the off-the-shelf satellite bus). Satellite mission designers must utilize individual mission constraints to decide at which point they wish to operate and which EP technology to use.

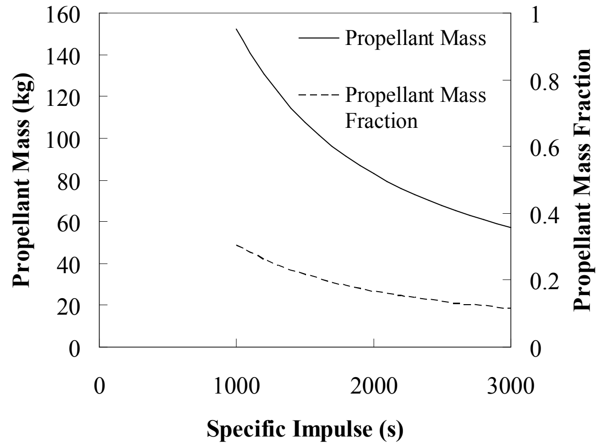

Figure 2. Propellant mass required for a LEO-to-MEO orbit change.

B. Plane Change

The next maneuver changes a satellite's orbit inclination by 90° at LEO. The same initial estimates apply to this maneuver as well, including a power range from 100 W to 1.5 kW and an I_{sp} range from 1,000 to 3,000 s.

Figure 3 shows the transfer time as a function of thruster power. The same general trends in Figure 1 apply to Figure 3; as thruster power increases or I_{sp} decreases, transfer time decreases. For a plane change of 90° with an initial satellite mass of 500 kg and a thruster efficiency of 0.5, the transfer time ranges from 314 to 23,280 days. The lowest calculated transfer time is 314 days, which is still very close to one year. Lower inclination changes will allow shorter transfer times, allowing possible mission use with EP.

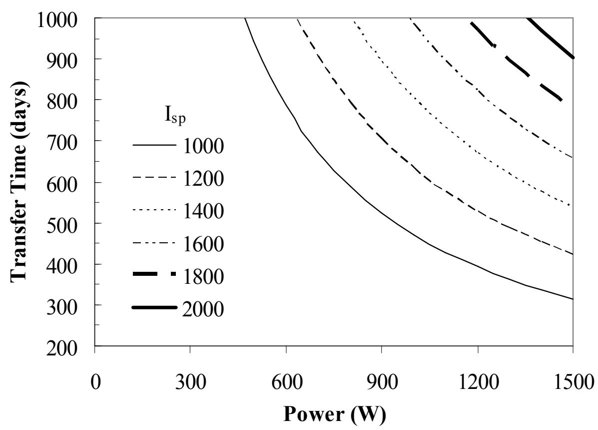

Figure 3. Transfer time associated with a 90° plane change at LEO.

However, missions that require large inclination changes do not seem to readily benefit from current EP technologies. This constraint identifies a future focus for low-power EP research.

To identify limits of EP use on such small satellite buses as those specified by the Air Force ORS program, this plane change study considers propellant masses from 0.5 to 0.7 with associated thruster I_{sp} to calculate power requirements. Some of these results are found feasible, while others are simply too high for small satellites. With a propellant mass of 0.5, the power required to achieve a 90° plane change in one year is approximately 5.58 kW with an I_{sp} of 2,700 s. At the higher limit, a propellant mass fraction of 0.65 requires 3.18 kW of power with an I_{sp} of 1,790 s, while a propellant mass fraction of 0.7 requires an input power of 2.59 kW with an I_{sp} of 1,560 s. A power level of 2.59 kW, and possibly 3.18 kW, is achievable with the addition of solar arrays to the satellite bus, but mission designers would need to consider whether this propellant mass fraction is too high for mission criteria.

Propellant mass is also important for satellite mission flexibility. Figure 4 shows propellant mass and propellant mass fraction as a function of thruster I_{sp}. Again, propellant mass is independent of thruster power, as seen in the analysis mathematics. Over the entire I_{sp} range of 1,000 to 3,000 s, the propellant mass varies between 232 and 423 kg. This is a significant portion of the gross satellite mass of 500 kg; the propellant mass fraction ranges from 0.46 to 0.85. Independently, these values will satisfy missions while adding tight constraints, but the relationship between the two is much more complicated.

In addition, Figure 4 shows three points which consider EP use with inclination changes of 10°, 20°, and 30°, I_{sp} of 1,500 s, and thruster power of 1 kW. Each change has a propellant mass fraction of 0.13, 0.24, and 0.34,

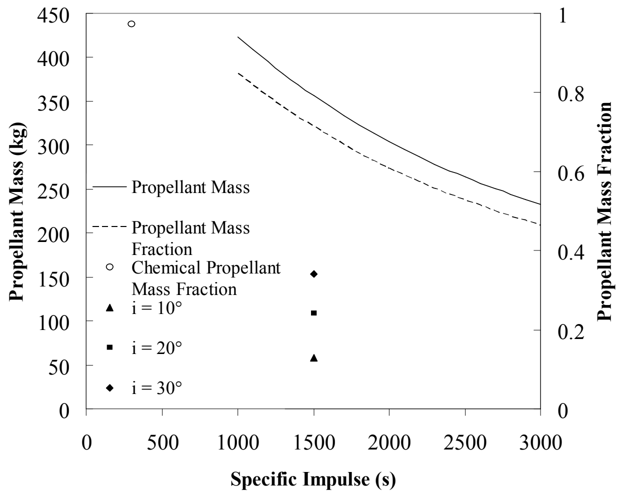

Figure 4. Propellant mass required for a 90° plane change at LEO. Orbit inclination is noted as *i*.

respectively. Small inclination changes show that EP use on proposed Air Force ORS programs would greatly benefit the mission by limiting the required propellant mass.

A comparison can be made at this point between chemical and EP performance. Chemical propulsion requires a ΔV of 10,538 m/s to complete a 90° plane change. Such chemical thrusters typically have an I_{sp} of 300 s. With the use of the rocket equation, Eq. (2), the propellant mass fraction for a chemical thruster changing a satellite's inclination by 90° is 0.972, as the point in Figure 4 shows. This high mass fraction leaves the final mass of a 500 kg satellite at approximately 14 kg. Based purely on required propellant mass, EP exceeds chemical propulsion in increasing the available payload mass.

An analysis of the transfer time for a 90° plane change shows that for transfer times less than one year, EP with an I_{sp} of 1,000 s would be the only choice of those thrusters considered in this analysis. If the I_{sp} level were reduced, such thrusters could achieve a shorter transfer time, but at the cost of increased propellant mass.

C. Phase Change

The final orbital maneuver changes the phasing of a satellite in orbit by 180°. Each initial estimate still applies to this analysis, including a power range of 100 W to 1.5 kW and I_{sp} range of 1,000 to 3,000 s.

Analysis includes propellant mass as a function of transfer time with varying I_{sp} and thruster power levels. Each analysis spans a transfer time of 0 to 100 hours with five different I_{sp} values. Figure 5a shows that a 100 W thruster rephases a satellite by 180° at a maximum propellant mass of 40 kg with a transfer time of 10 hours and at a minimum propellant mass of 0.7 kg with a transfer time of 100 hours. At the highest power level, Figure 5b shows that a 1.5 kW thruster rephases a satellite at a maximum propellant mass of 170 kg with a transfer time of approximately 3 hours and at a minimum propellant mass of 0.7 kg with a transfer time of 100 hours. During the analysis, it shows that above 50 hours of transfer time, the propellant mass for each power level is approximately the same. Above 40 hours this value changes slightly, but transfer times of less than 40 hours show large changes in propellant mass between thruster power levels.

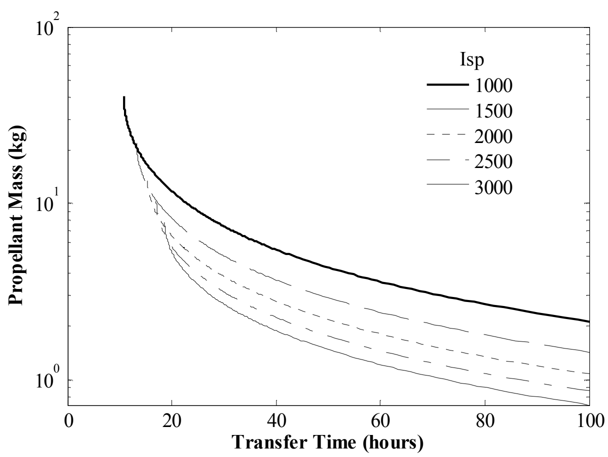

Figure 5a. Propellant mass required for a phase change of 180° with a 100 W thruster.

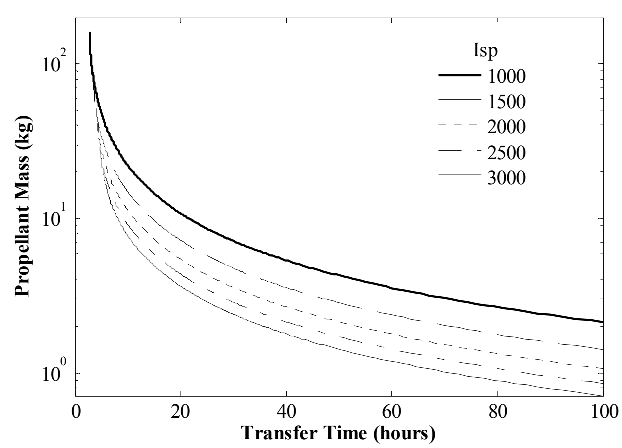

Figure 5b. Propellant mass required for a phase change of 180° with a 1.5 kW thruster.

At a maximum thruster power of 1.5 kW and an I_{sp} of 1,000 s, the propellant mass fraction is approximately 0.34 for the fastest transfer, while the lowest propellant mass fraction is approximately 0.0014 for a 100 hour transfer. These low propellant masses would allow larger payload masses in such readily available satellite buses as those desired by the Air Force ORS program. The transfer time associated with a 180° rephasing maneuver is within an appropriate limitation of 100 hours, or four days, at a minimum propellant mass.

IV. Conclusion

With the introduction of the Air Force ORS program, EP may provide the primary propulsive ΔV required for three orbital maneuvers: altitude changes, inclination changes, and phase changes Altitude changes from LEO-to-MEO are possible with any thruster characteristic of the I_{sp} and power ranges studied. However, assuming that a transfer time of more than one year is too long for a given mission, a minimum power requirement of 500 W for a 1,000 s I_{sp} thruster that consumes approximately 152 kg of propellant exists for a 500 kg satellite. From this point, any increase in thruster I_{sp} requires a large increase in thruster power to achieve a LEO-to-MEO altitude change under one year. At the higher limit, a 3,000 s I_{sp} thruster requires 1.5 kW of power and approximately 57 kg of propellant, a propellant mass fraction of 0.114, to achieve a one year LEO-to-MEO altitude change. Inclination changes as high as 90° are much more restrictive than the LEO-to-MEO altitude change. For a transfer time under one year, the only capable thruster has an I_{sp} of 1,000 s, requires at least 1 kW of power, and consumes approximately 423 kg of propellant, a propellant mass fraction of 0.85. In order to offer low-power EP advantages for large inclination changes, available power on commercial satellite buses, including solar array additions, must increase to 2.6 or 3.1 kW, with a thruster I_{sp} of 1,560 and 1,790 s, respectively, to fulfill the demands of one year transfer times. Satellite rephasing by 180° is the least expensive maneuver in terms of required propellant mass and input power for a 500 kg satellite. Propellant masses required for such satellite phasing with transfer times between 50 hours and 100 hours are found to be approximately the same for each I_{sp} under all power ranges. However, the propellant mass required for a 3 hour transfer with an input power of 1.5 kW is approximately 170 kg, a propellant mass fraction of 0.34, while a propellant mass of 0.7 kg, a propellant mass fraction of 0.0014, for any input power will achieve this transfer in 100 hours. Individual mission design requirements will appropriately constrain this range of transfer times and propellant masses.

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